

A Study of Free Convection Laminar Flow of an Incompressible Viscoelastic Fluid past a Porous Vertical Wall with Constant Suction

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ABSTRACT: The object of this problem is to study 'Free convection laminar flow of an incompressible visco-elastic fluid past a porous wall with constant suction'. Here an exact solution is obtained for the free convection laminar flow of an incompressible visco-elastic (Rivlin-Ericksen) fluid past a porous wall with constant suction. The variation in the rate of heat transfer with an increasing frequency is observed.

Nomenclature

| | |
|------------|---|
| k | Permeability parameter |
| g | Acceleration due to gravity |
| N | Nusselt number |
| P | Pressure |
| q | Heat flux per unit area |
| T | Temperature in the boundary layer |
| T_w | Temperature at the wall |
| T_∞ | Temperature of the free stream |
| t | Time |
| τ | Skin friction |
| u | Velocity in the x -direction |
| U_∞ | Velocity of the free stream |
| v | Velocity in y -direction |
| w | Velocity in z -direction |
| x, y | Coordinate system |
| X, Y, Z | Component of the body force in x, y and z directions respectively |
| ρ | The density |
| σ | Prandtl number |
| θ | Dimensionless temperature |
| ν | The kinematic cross viscosity |
| β | Coefficient of Kinematics visco-elasticity |

Formulation of the problem and governing equation

Here x -axis be taken along the vertical wall and y -axis at right angle to it. We have considered the free convection flow of an incompressible visco-elastic fluid past a porous vertical wall. The equations governing the flow are

$$\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial y} = g\beta_1(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial^2}{\partial y^2} \left(\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial y} \right) \quad (1)$$

$$\frac{\partial V}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + 2(2\beta + \nu) \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial v}{\partial y} = 0 \quad (3)$$



$$\frac{\partial T}{\partial t} + V \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} \tag{4}$$

Here β is the kinematic visco-elasticity, ν is the kinematic cross-viscosity and all other symbols have their usual meaning.

The continuity equation (3) shows that V is a function layer type. It is known that V must be negative, non-zero constant (V_ω). This restriction (Stuart) has also been taken while discussing the unsteady case.

Considering $V = -|V_\omega|$. (5)

Using the following non-dimensional quantities

$$y_1 = \frac{|V_\omega|y}{\nu} = \frac{y}{L}, \quad t_1 = -\frac{t}{L^2}, \quad u_1 = \frac{uL}{\nu}, \quad V_1 = \frac{VL}{\nu} = \frac{V}{|V_\omega|}$$

$T_1 = L^3 g \beta_1 (T - T_\infty) / \nu^2$ and $P_1 = \frac{PL^2}{\rho^2}$ into the equation (1) and (4) with the use of (5), we get the

following equation after dropping the suffixes

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = T_1 + \frac{\partial^2 u}{\partial y^2} + s \frac{\partial^2}{\partial y^2} \left(\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} \right) \tag{6}$$

and

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{\sigma} - \frac{\partial^2 T}{\partial y^2} \tag{7}$$

where $s = \beta / L^2$ a dimensionless quantity and σ is the Prandtl number.

The boundary conditions for the considered problem are

$$\begin{aligned} U = 0, \quad V = -|V_\omega| \text{ and } T = \theta(t) \text{ at } y = 0 \\ u = 1, \quad \frac{\partial u}{\partial y} = 0 \text{ and } T = 0 \text{ at } y = \infty \end{aligned} \tag{7}$$

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The boundary conditions for the considered problem are

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Solution of the Problem

For obtaining the solution of (6) and (7) satisfying the boundary conditions (8), we expand $U(y, t)$ and $T(y, t)$ in the following form

$$U(y, t) = U_0(y) + \varepsilon U_1(y) e^{i\omega t} \tag{9}$$

$$T(y, t) = T_0(y) + \varepsilon T_1(y) e^{i\omega t} \tag{10}$$

where ε may be taken as positive by suitable choice of the origin of time. Substituting (9) and (10) in equations (6) and (7) and equating the non-periodic and harmonic terms separately to zero, we get the following equations

$$sU''' - u_0'' - U_0' = T_0 \tag{11}$$

$$sU_1''' - (1 + i\omega s)U_1'' - U_1' + i\omega U_1 = T_1 \tag{12}$$

$$T_0'' + OT_0' = 0 \tag{13}$$

$$T_1'' + OT_1' = i\omega O \quad T_1 = 0 \tag{14}$$

The boundary conditions (8) reduce to



$$\begin{aligned}
 U_0(0) &= 0 = U_1(0), \\
 U_0(\infty) &= 1, U_1(\infty) = 0, \\
 U'_0(\infty) &= 0 = U'_1(\infty), \\
 T_0(0) &= \theta_0, T_1(0) = \theta_1, \\
 T_0(\infty) &= 0 = T_1(\infty).
 \end{aligned}
 \tag{15}$$

The solution of (13) and (14) subject to the boundary conditions (15) are as follows

$$\begin{aligned}
 T_0(y) &= \theta_0 e^{-\sigma y} \\
 T_1(y) &= \theta_1 e^{-hy}
 \end{aligned}
 \tag{16}$$

where $h = \frac{1}{2} \sigma (1 + \sqrt{1 + 4\omega / \sigma})$

the solution of (11) and (12) subject to condition (15) are

$$\begin{aligned}
 U_0(y) &= 1 - e^{-ay} + b(e^{-\sigma y} - e^{-ay}) \\
 U_1(y) &= C(e^{-hy} - e^{-h_1 y})
 \end{aligned}$$

where $s = \frac{1 - \sqrt{1 - 4s_0}}{2s_0}$, $s = -s_0$, $s_0 > 0$ ($0 \leq s_0 < 1/4$)

$$\begin{aligned}
 b &= \frac{\theta_0}{s_0 O^3 - O^2 + O} \\
 c &= -\frac{\theta_1}{-s_0 h^3 + (1 - \omega s_0) h^2 - h - \omega}
 \end{aligned}$$

and h_1 is the root of the following equation

$$s_0 h_1^3 + (1 - \omega s_0) h_1^2 + h_1 - \omega = 0
 \tag{17}$$

The temperature distribution is given by

$$T = \theta_0 e^{-\sigma y} + \varepsilon \theta_1 e^{-h_1 y + \omega t}
 \tag{18}$$

The velocity distribution inside the boundary layer is given by

$$u(y, t) = 1 - e^{-ay} + b(e^{-\sigma y} - e^{-ay}) + \varepsilon c(e^{-hy} - e^{-h_1 y}) e^{\omega t}
 \tag{19}$$

Heat Transfer and Skin Friction

The rate of heat transfer from the wall is given by

$$\begin{aligned}
 q &= -\frac{k V_w \nu^2}{g \beta_1 L^4} \left(\frac{\partial T}{\partial y} \right)_{y=0} \\
 &= \frac{k V_w \nu^2}{g \beta_1 L^4} (\theta_0 \sigma + \varepsilon \theta_1 h e^{\omega t})
 \end{aligned}
 \tag{20}$$

The Nusselt number is

$$N = \frac{L_q}{k(T_w - T_\infty)} = \frac{\theta_0 \sigma + \varepsilon \theta_1 h e^{\omega t}}{\theta_0 + \varepsilon \theta_1 e^{\omega t}}
 \tag{21}$$

Where, T_w is the wall temperature?

Conclusion

It is clear from the above expression that Nusselt number decreases as the frequency increases so the rate of heat transfer from the wall to the fluid decreases.

The expression for the skin friction is given by

$$\begin{aligned} \tau &= \frac{\mu^2 V_w}{L^2 \rho} \left(\frac{\partial u}{\partial y} \right)_{y=0} \\ &= \frac{\mu^2 V_w}{L^2 \rho} \left\{ a + (a - \sigma)b + \varepsilon c (h_1 - h e^{i\omega t}) \right\} \end{aligned} \quad (22)$$

In case of Newtonian fluid it takes the following form

$$\tau = \frac{\mu^2 V_w}{L^2 \rho} \left\{ 1 + (1 - \sigma)b' + \varepsilon c' (h_1' - h) e^{i\omega t} \right\} \quad (23)$$

where

$$\begin{aligned} b' &= \frac{\theta_0}{\sigma(1 - \sigma)} \\ c' &= -\frac{\theta_0}{h^2 - h - i\omega} \\ h' &= \frac{1 + \sqrt{1 + 4i\omega}}{2} \end{aligned}$$

It can be concluded that the wall stress is also altered from the classical case.

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